Reflective Elastic Scattering at LHC

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We discuss effects of reflective scattering for hadron and heavy nuclei collisions at the LHC and asymptotic energies. It is shown that the reflective scattering might lead to decreasing matter density with energy beyond the LHC energies. A limiting form of energy dependence of the hadron density is obtained. The unitarity upper bound for the absolute value of the real part of the elastic scattering amplitude and two-particle inelastic binary reaction amplitudes in impact parameter representation is two times less than the corresponding bound for the imaginary part of the elastic scattering amplitude. The former limit restricts a possible odderon contribution.

1 Reflective Scattering

A new physical interpretation of unitarity saturation in elastic scattering as a reflective scattering was proposed in [1] proceeding from optical analogy. This interpretation is related to the non-perturbative aspects of strong interactions and follows from the specific property of the unitarity saturation when the elastic S-matrix becomes negative and $S(s,b)|_{b=0} \to -1$ at $s \to \infty$. It should be noted that $S(s,b) = 1 + 2if(s,b)$, where $f(s,b)$ is the elastic scattering amplitude in the impact parameter representation.

In particular, we would like to note that the possible values of the elastic S matrix can be negative (in the pure imaginary case). Transition to the reflective scattering mode is naturally reproduced by the U-matrix form of elastic unitarisation. The elastic scattering S-matrix ($2 \to 2$ scattering matrix element) in the impact parameter representation is written in this unitarisation scheme in the form of a linear fractional transform:

$$S(s,b) = \frac{1 + iU(s,b)}{1 - iU(s,b)}, \tag{1}$$

where $U(s,b)$ is the generalised reaction matrix, which is considered to be an input dynamical quantity. For simplicity we consider the case of a pure imaginary U-matrix and make the replacement $U \to iU$ in (1). The reflective scattering mode ($S(s,b) < 0$) starts to appear at the energy s_R, which is determined as a solution of the equation $U(s_R, b = 0) = 1$. At $s > s_R$ the elastic scattering acquires ability for reflection, while the inelastic overlap function $h_{inel}(s,b)$ gets a peripheral impact parameter dependence in the region $s > s_R$. It should be noted that unitarity condition for the elastic scattering amplitude $F(s,t)$, which can be written in the form

$$\mathrm{Im}F(s,t) = H_{el}(s,t) + H_{inel}(s,t), \tag{2}$$

where $H_{el,inel}(s,t)$ are the corresponding elastic and inelastic overlap functions introduced by Van Hove [2]. The functions $H_{el,inel}(s,t)$ are related to the functions $h_{el,inel}(s,b)$ via the

Fourier-Bessel transforms, i.e.

$$H_{el,inel}(s,t) = \frac{s}{\pi^2}\int_0^\infty b\,db\,h_{el,inel}(s,b)J_0(b\sqrt{-t}). \tag{3}$$

The elastic and inelastic cross-sections can be obtained as follows:

$$\sigma_{el,inel}(s) \sim \frac{1}{s}H_{el,inel}(s,t=0). \tag{4}$$

Saturation of unitarity leads to the peripheral dependence of $h_{inel}(s,b)$. It is a manifestation of the self-damping of the inelastic channels at small impact parameters. The function $h_{inel}(s,b)$ reaches its maximum value at $b = R(s)$, note that

$$R(s) \sim \frac{1}{M}\ln s,$$

while an elastic scattering (due to reflection) occurs effectively at smaller values of impact parameter, i.e. $\langle b^2\rangle_{el} < \langle b^2\rangle_{inel}$. At the values of energy $s > s_R$ the equation $U(s,b) = 1$ has a solution in the physical region of impact parameter values, i.e. $S(s,b) = 0$ at $b = R(s)$. Fig. 1 shows the regions where elastic S-matrix has positive and negative values. Of course,

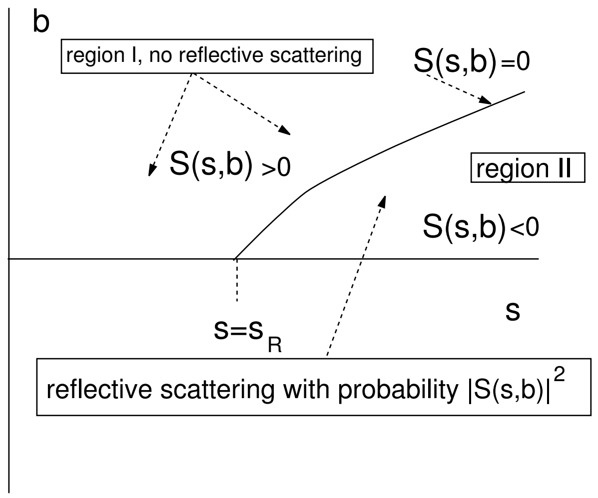

Figure 1: *Regions of positive (absorptive scattering) and negative values (absorptive and reflective scattering) of the function $S(s,b)$ in the s and b plane.*

the reflective scattering exists only in the elastic channel. For example, all inelastic binary reactions have amplitudes $\tilde{f}(s,b)$ in the impact parameter representation, which satisfy the inequality $|\tilde{f}(s,b)| \le 1/2$, while the elastic scattering amplitude satisfies $|f(s,b)| \le 1$. It follows from unitarity equation in the impact parameter representation:

$$\mathrm{Im} f(s,b) = h_{el}(s,b) + h_{inel}(s,b);$$

amplitudes $\tilde{f}(s,b)$ contribute to $h_{inel}(s,b)$. The upper bound for the elastic scattering cross-section is four times higher than the upper bound for the inelastic cross-section as it was recently demonstrated in [3]. Scattering dynamics in the elastic channel such as $pp \to pp$ is therefore strikingly different, e.g. from the inelastic binary diffractive process, such as $pp \to pN$, where N is an isobar. The latter reactions should have a peripheral impact parameter profile, which can be related to the dominating contribution of helicity-flip amplitudes. The unitarity limit and black disk limit are the same for the inelastic overlap function, but those limits are different for the elastic overlap function

$$h_{el}(s,b) \equiv |f(s,b)|^2.$$

The unitarity limit for the elastic overlap function is four times higher than the black disk limit. This is an important point for consideration of exclusive limit of inclusive reactions. Saturation of unitarity leads to suppression of the inelastic cross-section, i.e. at fixed impact parameter $(b < R(s))$ $h_{inel}(s,b) \to 0$ at $s \to \infty$ and

$$\sigma_{el}(s) \sim R^2(s), \quad \sigma_{inel}(s) \sim R(s). \tag{5}$$

Thus, $H_{el}(s,t)$ which has the following t-dependence

$$H_{el}(s,t) \sim \frac{RJ_1(R\sqrt{-t})}{\sqrt{-t}},$$

dominates over $H_{inel}(s,t)$, which depends on t like

$$H_{inel}(s,t) \sim RJ_0(R\sqrt{-t}),$$

at $-t = 0$, but it is not the case for the scattering in the non-forward directions. In this region these two functions have similar energy dependencies proportional to $R^{1/2}(s)$ at rather large fixed values of $-t$. The mean impact parameter values for elastic and inelastic interactions have also similar energy dependencies

$$\langle b^2 \rangle_{el}(s) \sim R^2(s), \quad \langle b^2 \rangle_{inel}(s) \sim R^2(s), \tag{6}$$

but the value of impact parameter averaged over all interactions (cf. [4])

$$\langle b^2 \rangle_{tot}(s) = \frac{\sigma_{el}(s)}{\sigma_{tot}(s)}\langle b^2 \rangle_{el}(s) + \frac{\sigma_{inel}(s)}{\sigma_{tot}(s)}\langle b^2 \rangle_{inel}(s)$$

acquires the main contribution from elastic scattering according to Eq. (5). Therefore, the inelastic intermediate states will give subleading contribution to the slope of diffraction cone $B(s)$,

$$B(s) \equiv \frac{d}{dt} \ln \left(\frac{d\sigma}{dt} \right) \Big|_{t=0},$$

at asymptotic energies. Indeed, since $B(s) \sim \langle b^2 \rangle_{tot}(s)$, it can be written in the form

$$B(s) = B_{el}(s) + B_{inel}(s),$$

where $B_{el}(s) \sim R^2(s)$, while $B_{inel}(s) \sim R(s)$. It should be noted that both terms $B_{el}(s)$ and $B_{inel}(s)$ are proportional to $R^2(s)$ in case of the absorptive scattering.

Under reflective scattering, the behaviour of the function $H_{inel}(s,t)$ is determined by a peripheral impact parameter profile and its $-t$ dependence is different. Meanwhile, the elastic overlap function $H_{el}(s,t)$ has similarities with that function in the case of approach where absorption is only presented. As a result, zeroes and maxima of the functions $H_{el}(s,t)$ and $H_{inel}(s,t)$ will be located at different values of $-t$ and zeroes and maxima of Im$F(s,t)$ will also be located at different position in the cases of absorptive and the reflective scattering. In the case of reflective scattering, dips and maxima will be located in the region of lower values of $-t$. We would like to note that the presence of reflective scattering enhances the large $-t$ region by factor $\sqrt{-t}$ compared to absorptive scattering. Despite that these two mechanisms lead at the asymptotics to the significant differences in the total, elastic and inelastic cross-section dependencies, their predictions for the differential cross-section of elastic scattering are not so much different at small and moderate values of $-t$.

2 Intermittent Remark on Unitarity and Real Part of Scattering Amplitude

It is evident that there are serious difficulties in accounting all known dynamical issues and limitations into a particular phenomenological model. But it is equally difficult to expect that the model inconsistent with unitarity (i.e. the one violating probability conservation law) would adequately reflect the dynamics of hadron interaction and provide reliable predictions. To fulfill unitarity condition under a model construction of the elastic amplitudes, it is natural to use unitarisation approaches such as eikonal or U-matrix, which consider amplitudes in the impact parameter space. They automatically guarantee that elastic amplitude in the impact parameter representation will obey unitarity condition.

Despite that the full implementation of unitarity is not possible nowadays (cf. e.g. [5]), the amplitude in the impact parameter space should not exceed unity anyway. However, when the amplitude $F(s,t)$ is constructed in the s and t representation, it is a priori not evident that the particular form of this amplitude being transformed into the impact parameter space $f(s,b)$ would satisfy unitarity. This remains to be true, even when the model under consideration leads to the predictions for observables which explicitly agree with axiomatic bounds, e.g. such as well known Froissart-Martin bound for the total cross-sections. Getting agreement with experimental data at finite energies and asymptotic bounds at $s \to \infty$ is not enough since a wide class of functional dependencies can successfully describe experimental data and provide correct asymptotic behaviour. Additional steps to justify that the impact parameter amplitude is *at least* less than unity in the whole region of kinematic variables are necessary.

In the above remarks we supposed that imaginary part of scattering amplitude is a dominating one. Further unitarity restriction exists for models which do not suppose domination of imaginary part of scattering amplitude, such as models with maximal odderon contribution [6].

Indeed, unitarity condition in the impact parameter representation for the elastic scattering amplitude can be rewritten in the form:

$$\mathrm{Im}f(s,b)[1-\mathrm{Im}f(s,b)] = [\mathrm{Re}f(s,b)]^2 + h_{inel}(s,b).$$

Since $0 \le \mathrm{Im}f(s,b) \le 1$, we obtain that unitarity limits the real part of scattering amplitude in the following way

$$[\mathrm{Re}f(s,b)]^2 \le 1/4,$$

$$-\frac{1}{2}\sqrt{1-4h_{inel}(s,b)} \le \mathrm{Re}f(s,b) \le \frac{1}{2}\sqrt{1-4h_{inel}(s,b)}.$$

The function $\mathrm{Re}f(s,b)$ can be sign changing one in contrast with $\mathrm{Im}f(s,b)$). This limitation, as it was already mentioned, is essential for the models with odderon and is indirectly in favour of the standard procedure of neglecting the real part of scattering amplitude compared to its imaginary part. It also is evident that absolute value of the real part and imaginary part of elastic scattering amplitude $f(s,b)$ cannot reach their maximal values simultaneously, moreover when $\mathrm{Im}f(s,b) \to 1$, saturating unitarity limit at large values of s in the region $b < R(s)$, then $\mathrm{Re}f(s,b) \to 0$ in this kinematic region. It should be noted that this saturation does not suppose that $\mathrm{Re}f(s,b)$ vanish everywhere. It means that $[\mathrm{Re}f(s,b)]^2$ should have a peripheral impact parameter profile. The same conclusion is valid when $\mathrm{Im}f(s,b) \to 1/2$, saturating the black disk limit at large values of s in the region $b < R(s)$, then $\mathrm{Re}f(s,b) \to 0$ because $h_{inel}(s,b) \to 1/4$ in this region. The above difference in the impact parameter profiles would result in the different energy dependencies of $\mathrm{Im}F(s,t=0)$ and $\mathrm{Re}F(s,t=0)$ bringing maximal odderon on the edge of contradiction with unitarity (or black disk) limit saturation. Of course, unitarity or black disk limits saturation itself does not follow from axiomatic field theory, but we would like to note, that it is much more natural to expect that it could be a manifestation of a maximal strength of strong interaction instead of behaviour of the real part of the forward scattering amplitude in the form $\mathrm{Re}F(s,t=0) \sim s\ln^2 s$ as it happens in the models incorporating the maximal odderon regime.

3 Reflective Scattering and Deconfinement

Possible existence of the reflective scattering at very high energies implies that confinement becomes stronger and stronger as the collision energy increases and proton collisions resemble more and more collisions of hard spheres. In this section we address one aspect of the broad problem of transition to the deconfined state of matter, namely, we discuss the role of the reflective scattering on the energy dependence of density in the percolation mechanism of the transition to the deconfined state of matter.

The main idea of the percolation mechanism of deconfinement is a formation in the certain volume of a connected hadron cluster due to increasing temperature and/or hadron density [7], i.e. when vacuum as a connected medium disappears, the deconfinement takes place. This process has typical critical dependence on particle density. Thus, it was proposed to use percolation to define the states of matter and consider the disappearance of a large-scale vacuum as the end of hadronic matter existence [7, 8].

The probability of reflective scattering at $b < R(s)$ and $s > s_R$ is determined by the magnitude of $|S(s,b)|^2$; this probability is equal to zero at $s \le s_R$ and $b \ge R(s)$ (region I in Fig. 1). At the energies $s > s_R$ reflective scattering will mimic the presence of a repulsive core in hadron and meson interactions. Presence of the reflective scattering can be accounted for using van der Waals method (cf. [9]). This approach was used originally for description of the fluids behaviour starting from the gas approximation by means of taking into account the nonzero size of molecules. Consider central collision of two identical nuclei having N hadrons in total with centre-of-mass energy $\sqrt{s}$ per nucleon and calculate hadron density $n_R(T,\mu) = N/V$ in the initial state at given temperature T and baryochemical potential μ in the presence of reflective scattering. The effect of the reflective scattering of hadrons is equivalent to decrease of the volume of the available space which the hadrons are able to occupy in the case when reflective scattering is absent. Thus following to van der Waals method, we must then replace volume V by

$$V - p_R(s)V_R(s)\frac{N}{2},$$

i.e. we should write

$$n(T,\mu) = \frac{N}{V - p_R(s)V_R(s)\frac{N}{2}},$$

where $n(T,\mu)$ is hadron density without account for reflective scattering and $p_R(s)$ is the averaged over volume $V_R(s)$ probability of reflective scattering:

$$p_R(s) = \frac{1}{V_R(s)}\int_{V_R(s)} |S(s,r)|^2 d^3x.$$

The volume $V_R(s)$ is determined by the radius of the reflective scattering. Here we assume spherical symmetry of hadron interactions, i.e. we replace impact parameter b by r and approximate the volume $V_R(s)$ by $V_R(s) \simeq (4\pi/3)R^3(s)$. Hence, the density $n_R(T,\mu)$ is connected with corresponding density in the approach without reflective scattering $n(T,\mu)$ by the following relation

$$n_R(T,\mu) = \frac{n(T,\mu)}{1+\alpha(s)n(T,\mu)},$$

where $\alpha(s) = p_R(s)V_R(s)/2$. Let us now estimate change of the function $n_R(T,\mu)$ due to the presence of reflective scattering. We can approximate $p_R(s)$ by the value of $|S(s,b=0)|^2$ which tends to unity at $s \to \infty$. It should be noted that the value $\sqrt{s_R} \simeq 2\,\mathrm{TeV}$ [10]. Below this energy there is no reflective scattering, $\alpha(s) = 0$ at $s \leq s_R$, and therefore corrections to the hadron density are absent. Those corrections are small when the energy is not too much higher than s_R. At $s \geq s_R$ the value of $\alpha(s)$ is positive, and presence of reflective scattering diminishes hadron density. We should expect that this effect would already be noticeable at the LHC energy $\sqrt{s} \simeq 5\,\mathrm{TeV}$ in Pb+Pb collisions. At very high energies ($s \to \infty$)

$$n_R(T,\mu) \sim 1/\alpha(s) \sim M^3/\ln^3 s.$$

This limiting dependence for the hadron density appears due to the presence of the reflective scattering which results in similarity of head-on hadron collisions with scattering of hard spheres. It can be associated with saturation of the Froissart-Martin bound for the total cross-section. It should be noted that this dependence has been obtained under assumption on spherical symmetry of hadron interaction region. Without this assumption, limiting dependence of the hadron density in transverse plane can only be obtained, i.e. transverse plane density of hadrons would have then the following behaviour

$$n_R(T,\mu) \sim M^2/\ln^2 s.$$

To conclude this section, we would like to note that the lower densities of hadron matter are needed for percolation (and transition to the deconfined state) in the presence of reflective scattering. It might be useful to note that the rescattering processes are also affected by the reflective scattering. Reflective scattering would lead to noticeable effects at the LHC energies and beyond and could help in searches of the deconfined state and studies of properties of transition mechanism to this state of matter which might proceed by means of percolation. Thus, it will affect description of initial state dynamics in nuclear interactions at the LHC energies by introducing notion of limiting density of strongly interacting matter at respective energies.

4 Conclusion

Thus, at very high energies there would be two different regions of impact parameter distances in particles collisions, namely the outer region (peripheral collisions) where elastic scattering has exclusively a shadow origin and inner region (central collisions) where reflective and absorptive scattering give competing contributions, reflective scattering contribution increases while absorptive scattering contribution decreases at fixed impact parameter. It is not surprising that the model with reflective scattering contribution leads to significantly higher values for total and elastic cross-sections at the LHC energies[1] while it renders the standard values for the inelastic cross-section. In the geometric terms, the generic scattering picture at fixed energy beyond the black disc limit can be described as a scattering off a partially reflective and partially absorptive disk surrounded by the black ring (which becomes grey at larger values of the impact parameter). The evolution with energy is characterised by increasing albedo due to the interrelated increase of reflection and decrease of absorption at small impact parameters. This picture predicts that the scattering amplitude at the LHC energies is beyond the black disk limit at small impact parameters and it provides explanation for the regularities observed in cosmic rays studies, e.g. the existence of the knee in the cosmic rays spectrum. It leads also appearance of limiting density dependent on energy which takes place only at very high energies and has an origin related to unitarity saturation.

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